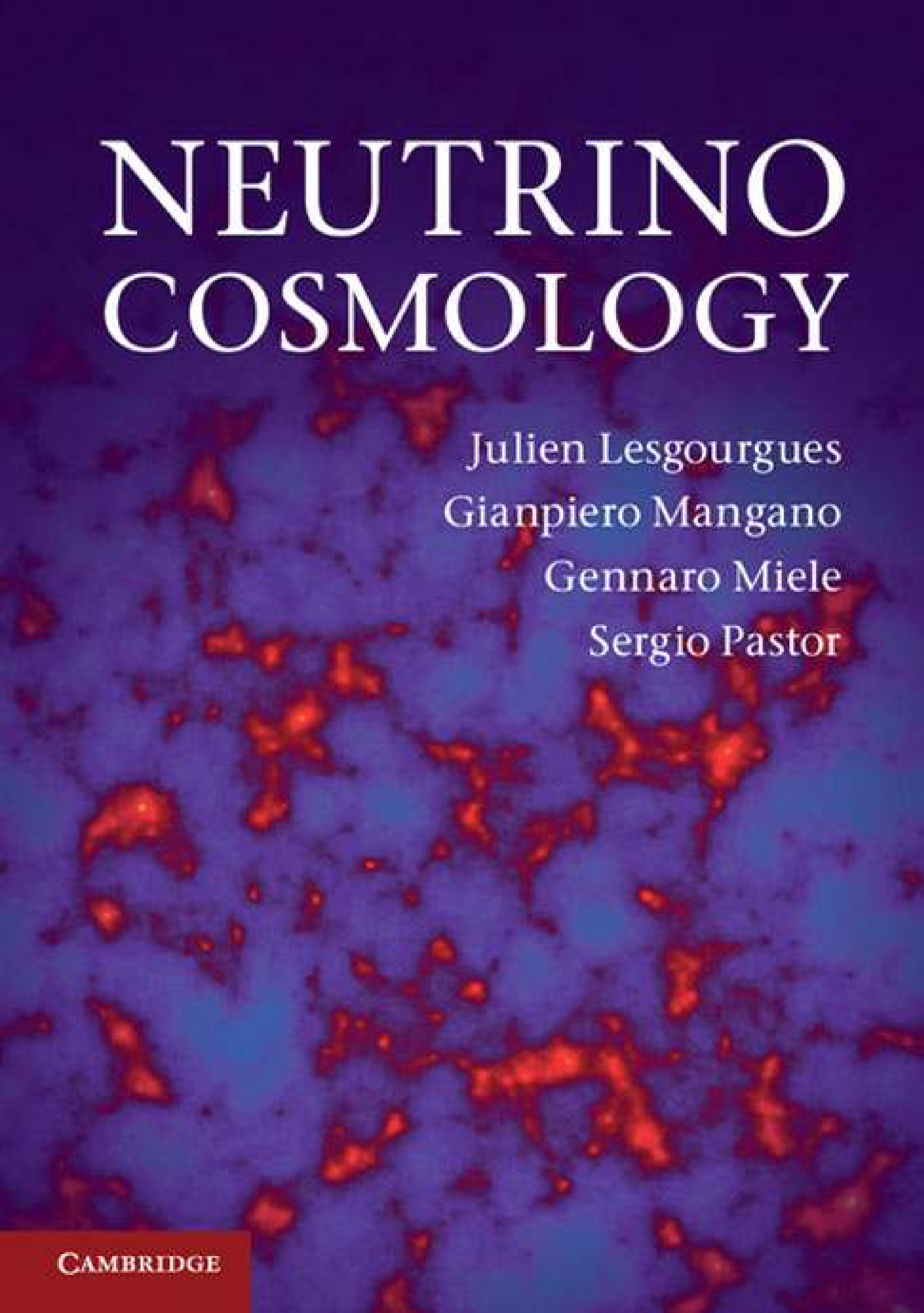
NEUTRINO
COSMOLOGY
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NEUTRINO COSMOLOGY

The role that neutrinos have played in the evolution of the Universe is one of the most fascinating research areas that has stemmed from the interplays between cosmology, astrophysics and particle physics. In this self-contained book, the authors bring together all aspects of the role of neutrinos in cosmology, spanning from leptogenesis to primordial nucleosynthesis, and from their role in CMB and structure formation to the problem of their direct detection.

The book starts by guiding the reader through aspects of fundamental neutrino physics, the standard cosmological model and statistical mechanics in the expanding Universe, before discussing the history of neutrinos in chronological order from the very early stages until today. This timely book will interest graduate students and researchers in astrophysics, cosmology and particle physics, who work with either a theoretical or experimental focus.

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NEUTRINO COSMOLOGY

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Preface

To Arianna, Carmen, Isabelle, and María José

When neutrinos first came on the scene in 1930, their father, Wolfgang Pauli, confessed to his colleague, the astronomer Walter Baade, that to save energy conservation in β-decays (quoted in Hoyle, 1967),

I have done a terrible thing today, something which no theoretical physicist should ever do. I have suggested something that can never be verified experimentally.

This was perhaps the only time Pauli was mistaken. Less than 30 years later, neutrinos were discovered by Reines and Cowan.

Since then, we have learned so many things about neutrinos that Pauli himself would be very surprised. More than this, understanding neutrino properties has always brought new insights into the whole field of fundamental interactions, and new theoretical paradigms.

Today we know quite accurately how to describe their feeble interactions with matter, from the very first attempts of Fermi to the succesful Standard Model of electroweak interactions. Many pieces of information have been collected in laboratory experiments, the traditional setting of particle physics. The study of neutrino interactions has been pursued at accelerators and reactors and, more recently, by sending neutrino beams produced at accelerators to underground laboratories. Accelerator experiments have also confirmed that there are only three generations of light neutrinos which are weakly interacting.

The main breakthrough in neutrino physics over the last few decades came from a different environment: astrophysics. The solar neutrino problem – an observed deficit of neutrino flux from the sun – along with the atmospheric neutrino anomaly, has led to the discovery that neutrinos are massive particles. We do not understand their mass spectrum yet, nor why they are such light particles. On the experimental side, remarkable improvements are expected in the next few years, both in measuring the neutrino mass scale using tritium β-decay, and in understanding the real *nature* of neutrinos as Dirac or Majorana particles. At the same time, intense theoretical activity is going on, addressing the neutrino mass problem, which is seen as a possible clue to unveiling the behaviour of fundamental interactions at high energy scales.

The most spectacular property of neutrinos is deeply rooted in quantum mechanics. As first suggested by Bruno Pontecorvo, neutrinos exhibit oscillations among different flavours during their time evolution, leading to an incredibly rich phenomenology. The parameters characterizing this oscillatory behaviour are currently quite well known, because of the interplay of a variety of different experimental techniques and neutrino sources of both terrestrial and astrophysical origin. Just at the time of writing this book, the last missing piece of the puzzle, one of the neutrino mixing angles, was measured with good precision. No doubt neutrino oscillation physics will represent a leading research line in the coming years.

Since the early works on the synthesis of light nuclei in the Big Bang model in the late forties, it was clear that neutrinos are not simply passive spectators during the expansion of the universe. Through their weak interactions with other particles, as well as their *gravity*, they influence a wide variety of phenomena which took place in the early stages of the life of the universe, till very recent epochs. This means that using observations of astrophysical quantities, related to specific phases of the expansion history, we have a further way to constrain neutrino properties at different energy, time and length scales, which in some cases are not accessible to laboratory experiments. Just to give a few examples, some constraints on the number of light weakly interacting neutrinos and on their mass scale were obtained using observations of primordial ^{4}He and of the total energy density of the universe well before laboratory experiments could provide comparable information.

This vast arena in which to test neutrino properties is usually referred to as *neutrino cosmology*, and that is what this book is about. By its very nature, it is a multidisciplinary reasearch field, where the different expertises and backgrounds of theoretical and experimental particle physicists, astrophysicists and cosmologists find a meeting point and a common language. It is a branch of an even broader scientific activity, commonly called *astroparticle physics*, aimed at understanding fundamental interactions by exploiting observations of very large objects, such as astrophysical sources, or the universe as a whole.

In the last two decades, we have witnessed a big boost along this research line, due to outstanding improvements in the number and quality of astrophysical observations. Large galaxy surveys, detailed maps of the cosmic microwave background, observations of primordial nuclear abundances and new ways to trace the expansion history of the universe are just a few examples of this experimental effort. Whereas only 20 years ago neutrino cosmology was in its infancy, and theoretical physicists were typically satisfied by order-of-magnitude calculations made on the back of the envelope, the situation has changed rapidly since then. Observations currently require a much more detailed analysis, and provide several new tests of theoretical models.

This book is a summary of the history of the universe from a neutrino perspective. The first two chapters introduce the three important theoretical tools that will be widely used in the following: the basics of neutrino interactions and properties in the framework of the Standard Model of particle physics, the homogeneous and isotropic cosmological model, and some concepts of kinetic theory. We have done our best to present a pedagogical and self-contained discussion of these topics, and we are not sure that we have succeeded in this respect. Indeed, the subject of this book is intrinsically multidisciplinary, and covering all topics in a detailed and self-consistent manner – while keeping the number of pages reasonable – was a major challenge. Readers who are not familiar with, say, quantum field theory, general relativity, gauge issues in cosmology or the theory of inflation will need further reading in more specialized books or reviews. In any case, we tried at least to introduce all the concepts that are necessary for understanding the remaining chapters. We hope that this part may also trigger the reader's interest in further studying the topics he or she might be unfamiliar with. To this end, we give a long list of possible references.

The remaining chapters are devoted to different aspects of the role of neutrinos in cosmology, in *chronological order*, as they intervene in the evolution of the universe, from the very early stages till today. Chapter 3 addresses the issue of baryogenesis, the dynamical production of the baryon asymmetry observed in our universe, and in particular a scenario deeply related to neutrino properties, called leptogenesis. Chapter 4 deals with the dynamics of neutrino oscillations in a cosmological setting, and also with primordial nucleosynthesis, one of the main pillars of the cosmological model, providing a lot of information about neutrino physics. Chapter 5 explains the properties of cosmic microwave background anisotropies, which contain a huge quantity of information about the whole history of the universe, and shows how they are impacted by neutrinos. Chapter 6 describes the dynamics of structure formation on very large scales – those of galaxies, clusters, etc. – which is crucially affected by the abundance, mass and properties of neutrinos. Finally, Chapter 7 presents a summary of the methods which have been proposed so far to detect the relic neutrino background in the laboratory, and a brief discussion of the anisotropies that such detectors would see if they could ever become operational. As we will see, this is a very challenging task, the ultimate dream of a *neutrino cosmologist*.

In this book we will adopt the signature $(-+++)$, except in Chapters 1 and 4, which are more particle physics oriented, where we adopt the more widely used $(+---)$. Unless otherwise mentioned, we use natural units.

If we look into some of the available Web archives for scientific papers related to both neutrinos and cosmology, the query will return a number of publications of the order of several thousands. This gives an idea of how intense the activity is in this research field. In the following pages, the reader will not find a complete analysis of all possible models and ideas proposed so far. Some alternatives to the mainstream scenarios – sometimes extremely interesting and intriguing – have not been considered in our discussion, and are cited in our (rather long) list of references. We apologize for all omissions. However, in writing this book, our guideline has been to try to present the main physical aspects of the phenomena neutrinos are involved in, rather than to go through all their possible variations. In a sense, what we have mainly considered is a *standard* neutrino cosmology, describing what is currently well established on solid theoretical and experimental bases. We hope this might be helpful for students and researchers who are interested in approaching this fascinating research field, starting from different cultural backgrounds. If this ambitious goal is achieved even partially, we will be happy with our contribution to a process that is well on the way, namely, the emergence of a homogeneous community of theoretical and experimental particle physicists, cosmologists and astrophysicists.

This book is the result of the authors' friendship over many years. However, it would not have been written were not for enlightening discussions and collaborations with many of our colleagues. Several topics that the reader hopefully will find interesting in the following pages are the outcome of their work and enthusiasm, and of their sharing with us their knowledge and experience.

We warmly thank Benjamin Audren, Steve Blanchet, Diego Blas, Alexei Boyarski, Marco Cirelli, Gaëlle Giesen, Martin Hirsch, Michal Malinský, Oleg Ruchayskiy, Pasquale Serpico, Mikhail Shaposhnikov and Mariam Tórtola for reading a draft version of this book.

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1

The basics of neutrino physics

Like the actors in ancient Greek tragedy and comedy, neutrinos play more than one role in the drama of the expanding universe. They couple to gravity and contribute to Einstein equations which rule the expansion dynamics. Furthermore, they interact in the primordial plasma with charged leptons and hadrons via electroweak interactions, until the rates for these processes become so low compared with the typical expansion rate that they *decouple* and start to propagate freely along geodesic lines. Any quantitative description of their role in cosmology thus requires several inputs from the theory of fundamental interactions, as well as a knowledge of their basic properties, such as masses and, in some cases, the features of neutrino flavour oscillations.

Neutrino interactions have been well understood since the first theory of β-decay proposed by Enrico Fermi in 1934, and now are succesfully and beautifully described by the unified picture of electroweak interactions. In the low energy limit the strength of these interactions is encoded in a single coupling, the Fermi coupling constant $G_{\rm F}$, whose value, combined with the Newton constant, fixes the time of neutrino decoupling. From the strong experimental evidence in favour of neutrino oscillation, we also know that neutrinos are massive particles, and this, as we will see at length in the following, has a strong impact on how structures, i.e., inhomogeneities in the universe, grow on certain length scales.

As a viaticum for this journey in the land of neutrino cosmology, it seemed worth-while to the authors to provide the reader with certain minimal information on the basic properties of neutrinos, both at the level of their theoretical formulation and from the experimental point of view.

Unfortunately, to keep a self-contained summary of these topics reasonably short requires the reader to be acquainted with the basics of quantum field theory and of the gauge principle, which are treated in full detail in many excellent textbooks (e.g., Itzykson and Zuber, 1980; Halzen and Martin, 1984; Weinberg, 1995; Peskin and Schroeder, 1995). In case he or she is familiar with neutrino physics, it is then possible to skip this chapter, though it might be useful to go through it anyway to become familiar with our notation. For all other readers, the following sections can represent only a too-brief synthesis of the present understanding of neutrino properties, hopefully sufficient for them to comfortably read the rest of this book, and likewise hopefully to trigger their curiosity for a deeper understanding of neutrino physics.

Here is a summary of this introductory chapter. After a short review of the Standard Model of fundamental interactions, which covers only the details of its electroweak sector, we describe the main observable properties of neutrinos – interaction processes, Dirac and Majorana masses and flavour oscillations – including a summary of bounds on a certain class of exotic interactions which are beyond our present understanding of microscopic physics but are typically predicted by extensions of the Standard Model which represent its *ultraviolet completion*. We then give a résumé of experimental results on flavour oscillation experiments, laboratory neutrino mass bounds and neutrinoless double-β decay, the last being an *experimentum crucis* to test their Dirac or Majorana nature.

1.1 The electroweak Standard Model

Gauge symmetry has proven to be a powerful guideline to building up a satisfactory theory of fundamental interactions. Strong and electroweak interactions are described by a relativistic quantum field theory based on the gauge symmetry principle for the group $SU(3)_C \times SU(2)_L \times U(1)_Y$, where C, L and Y denote *colour*, *left-handed chirality* and *weak hypercharge* (Glashow, 1961; Weinberg, 1972; Salam, 1968; Fritzsch *et al.*, 1973; Gross and Wilczek, 1973; Politzer, 1973; Weinberg, 1973). The model is so successful that it is now usually referred to as the 'Standard Model' (SM) of elementary particles.

Whereas the strong sector $SU(3)_C$ symmetry remains unbroken, and hence is an exact symmetry at any energy level, the electroweak forces undergo spontaneous symmetry breaking via the Higgs mechanism, which reduces the symmetry of the model at low energies to $SU(2)_L \times U(1)_Y \to U(1)_Q$, with Q being the electric charge. In the following we will focus our attention on the electroweak sector only, because neutrinos, like all leptons, do not carry colour charge and are not strongly interacting.

The requirement that a field theory is gauge-invariant under a particular symmetry group strictly fixes the form of the interaction and the number of gauge bosons. Unfortunately, it leaves quite a high level of arbitrariness in the choice of the irreducible representations (IR) of the gauge group to accommodate fermions and the Higgs scalar bosons. The only constraint is provided by the cancellation of

Table 1.1 *Elementary fermions in the SM*

Generation	1st	2nd	3rd
quarks	u	c	t
	d	s	b
leptons	ν_e	ν_μ	ν_τ
	e^-	μ^-	τ^-

Table 1.2 *Electroweak quantum numbers of fermions in the SM*

Fermion IRs under $SU(2)_L \times U(1)_Y$			I	I_3	Y	Q
$L_{eL} \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$L_{\mu L} \equiv \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$L_{\tau L} \equiv \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1/2	$1/2$ $-1/2$	-1	0 -1
$l_{eR} \equiv e_R$	$l_{\mu R} \equiv \mu_R$	$l_{\tau R} \equiv \tau_R$	0	0	-2	-1
$Q_{1L} \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$Q_{2L} \equiv \begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$Q_{3L} \equiv \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	1/2	$1/2$ $-1/2$	1/3	$2/3$ $-1/3$
$q^U_{uR} \equiv u_R$ $q^D_{dR} \equiv d_R$	$q^U_{cR} \equiv c_R$ $q^D_{sR} \equiv s_R$	$q^U_{tR} \equiv t_R$ $q^D_{bR} \equiv b_R$	0	0	$4/3$ $-2/3$	$2/3$ $-1/3$

the chiral anomaly, a condition which must be fulfilled if gauge symmetry should also be respected at the quantum level.

The currently known fermionic elementary particles (spin $s = 1/2$) are split into three generations of *quarks* and *leptons*; see Table 1.1. Each generation of fermions is described by the IRs of the electroweak gauge group as shown in Table 1.2, where we also report their charges.

By I we denote the weak isospin, which is 1/2 for $SU(2)_L$ doublets and 0 for singlets, respectively, whereas I_3 is its third component. The electric charge Q is given by the Gell-Mann–Nishijima relation $Q = I_3 + Y/2$.

The three-generation electroweak Lagrangian density is

$$\begin{aligned}
\mathcal{L} = {} & i\overline{L'_{\alpha L}}\not{D}L'_{\alpha L} + i\,\overline{Q'_{\alpha L}}\not{D}Q'_{\alpha L} + i\overline{l'_{\alpha R}}\not{D}l'_{\alpha R} \\
& + i\overline{q'^D_{\alpha R}}\not{D}q'^D_{\alpha R} + i\overline{q'^U_{\alpha R}}\not{D}q'^U_{\alpha R} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& + \left(D_\rho\Phi\right)^\dagger(D^\rho\Phi) + \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2
\end{aligned}$$

Table 1.3 *Electroweak quantum numbers of the Higgs doublet*

Higgs doublet	I	I_3	Y	Q
$\Phi(x) \equiv \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	1/2	1/2	+1	1
		−1/2		0

$$
\begin{aligned}
&- \left(Y'^{l}_{\alpha\beta}\, \overline{L'_{\alpha L}}\, \Phi\, l'_{\beta R} + Y'^{l*}_{\alpha\beta}\, \overline{l'_{\beta R}}\, \Phi^\dagger\, L'_{\alpha L} \right) \\
&- \left(Y'^{D}_{\alpha\beta}\, \overline{Q'_{\alpha L}}\, \Phi\, q'^{D}_{\beta R} + Y'^{D*}_{\alpha\beta}\, \overline{q'^{D}_{\beta R}}\, \Phi^\dagger\, Q'_{\alpha L} \right) \\
&- \left(Y'^{U}_{\alpha\beta}\, \overline{Q'_{\alpha L}}\, (i\sigma_2\Phi^*)\, q'^{U}_{\beta R} + Y'^{U*}_{\alpha\beta}\, \overline{q'^{U}_{\beta R}}\, (-i\Phi^T\sigma_2)\, Q'_{\alpha L} \right),
\end{aligned} \tag{1.1}
$$

where Φ is the Higgs doublet, whose properties are reported in Table 1.3. σ_2 is a Pauli matrix, and α is the generation index. In the following, repeated indices are summed over, unless differently specified. The covariant derivative D_μ is defined as

$$
D_\mu \equiv \partial_\mu + ig\vec{A}_\mu \cdot \frac{\vec{\sigma}}{2} + ig' B_\mu \frac{Y}{2}, \tag{1.2}
$$

with $\vec{A}^\mu \equiv (A_1^\mu, A_2^\mu, A_3^\mu)$ and B^μ denoting the gauge boson fields of the $SU(2)_L$ and $U(1)_Y$ factors.

The canonical kinetic (and self-interacting for the $SU(2)_L$ factor) term for gauge bosons is written in terms of the electroweak tensors $\vec{F}^{\mu\nu} \equiv (F_1^{\mu\nu}, F_2^{\mu\nu}, F_3^{\mu\nu})$ and $B^{\mu\nu}$, where

$$
\begin{aligned}
F_a^{\mu\nu} &= \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g \sum_{b,c=1}^{3} \varepsilon_{abc} A_b^\mu A_c^\nu \\
B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu.
\end{aligned} \tag{1.3}
$$

In the expression (1.1), fermion fields are marked by a *prime* to denote that these fields in general are not mass eigenstates, as will be discussed in detail in the next sections. Equation (1.1) contains in the first two lines the kinetic and electroweak interaction terms for leptons and quarks and the pure gauge boson term, whereas the third line accounts for the Higgs sector responsible for the symmetry breaking. Finally, the last three lines correspond to the Yukawa terms characterized by the complex couplings $Y'^{l}_{\alpha\beta}$, $Y'^{D}_{\alpha\beta}$ and $Y'^{U}_{\alpha\beta}$. They are responsible for charged leptons and quark masses and mixing.

We note that in its minimal version, there are no right-handed neutrino states ν_R in the SM, which would be a singlet under all symmetry group factors. This implies that *active* neutrinos ν_L remain massless, because there are no mass terms which appear as a consequence of symmetry breaking, differently from charged leptons and quarks. The extension of the model to massive neutrinos will be discussed in the following.

From the first two lines of Eq. (1.1) one can extract the charged-current and neutral-current weak interaction Lagrangian densities, denoted by $\mathcal{L}_I^{(CC)}$ and $\mathcal{L}_I^{(NC)}$. In particular, one gets

$$\mathcal{L}_I^{(CC)} = -\frac{g}{2\sqrt{2}} J_W^\mu \, W_\mu + \text{h.c.}, \tag{1.4}$$

where $J_W^\mu = J_{W,L}^\mu + J_{W,Q}^\mu$ and

$$\begin{aligned} J_{W,L}^\mu &= 2\left(\overline{\nu'_{eL}}\gamma^\mu e'_L + \overline{\nu'_{\mu L}}\gamma^\mu \mu'_L + \overline{\nu'_{\tau L}}\gamma^\mu \tau'_L\right) \\ J_{W,Q}^\mu &= 2\left(\overline{u'_L}\gamma^\mu d'_L + \overline{c'_L}\gamma^\mu s'_L + \overline{t'_L}\gamma^\mu b'_L\right). \end{aligned} \tag{1.5}$$

The gauge boson field $W^\mu \equiv (A_1^\mu - i A_2^\mu)/\sqrt{2}$ by definition annihilates a W^+ boson and creates a W^- boson. The neutral current density

$$\mathcal{L}_I^{(NC)} = -\frac{g}{2\cos\theta_W} J_Z^\mu \, Z_\mu + \text{h.c.}, \tag{1.6}$$

where $J_Z^\mu = J_{Z,L}^\mu + J_{Z,Q}^\mu$ and

$$\begin{aligned} J_{Z,L}^\mu &= 2\left(g_L^\nu \overline{\nu'_{\alpha L}}\gamma^\mu \nu'_{\alpha L} + g_L^l \,\overline{l'_{\alpha L}}\gamma^\mu l'_{\alpha L} + g_R^l \,\overline{l'_{\alpha R}}\gamma^\mu l'_{\alpha R}\right) \\ J_{Z,Q}^\mu &= 2\left(g_L^U \,\overline{q'^U_{\alpha L}}\gamma^\mu q'^U_{\alpha L} + g_R^U \,\overline{q'^U_{\alpha R}}\gamma^\mu q'^U_{\alpha R} + g_L^D \,\overline{q'^D_{\alpha L}}\gamma^\mu q'^D_{\alpha L} + g_R^D \,\overline{q'^D_{\alpha R}}\gamma^\mu q'^D_{\alpha R}\right). \end{aligned} \tag{1.7}$$

The gauge field Z^μ is defined *via* the rotation $Z^\mu = \cos\theta_W \, A_3^\mu - \sin\theta_W B^\mu$, where $\tan\theta_W = g'/g$ and $e = g\sin\theta_W$. Finally, the couplings $g_L^{\nu,l,U,D}$ and $g_R^{l,U,D}$ are given by the relations

$$g_L^f = I_3^f - Q_f \sin^2\theta_W \tag{1.8}$$

$$g_R^f = -Q_f \sin^2\theta_W, \tag{1.9}$$

which are summarized in Table 1.4.

1.2 Spontaneous symmetry breaking and fermion masses

It is easy to see that a naive mass term such as $\overline{e_L} e_R + \text{h.c.}$ is not allowed in the Lagrangian density (1.1) because it would spoil the symmetry invariance under the gauge group $SU(2)_L \times U(1)_Y$. However, when this group symmetry is broken,

Table 1.4 *Neutral-current couplings for the elementary fermions*

g_L	g_R
$g_L^\nu = \frac{1}{2}$	
$g_L^l = -\frac{1}{2} + \sin^2\theta_W$	$g_R^l = \sin^2\theta_W$
$g_L^U = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W$	$g_R^U = -\frac{2}{3}\sin^2\theta_W$
$g_L^D = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$	$g_R^D = \frac{1}{3}\sin^2\theta_W$

masses are produced via the celebrated Higgs–Englert–Brout–Guralnik–Hagen–Kibble mechanism (Englert and Brout, 1964; Guralnik *et al.*, 1964; Higgs, 1964a,b).

The dynamics of the Higgs field Φ is ruled by the term in $\mathcal{L}$

$$\mathcal{L}_\mathrm{H} = \left(D_\rho \Phi\right)^\dagger \left(D^\rho \Phi\right) - V(\Phi) = \left(D_\rho \Phi\right)^\dagger \left(D^\rho \Phi\right) + \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2 . \tag{1.10}$$

In quantum field theory, the minimum of the potential defines the ground state around which the fields are expanded in terms of creation and annihilation operators. Quantum excitations on the ground state correspond to particle states. Note that only neutral fields with vanishing spin (scalar) may have nontrivial ground states; otherwise this would spoil the electric charge conservation and the invariance under spatial rotations. The Higgs field ground state value $\langle\Phi\rangle$, hereafter referred to as the *vacuum expectation value* (vev), can be written in the form

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{1.11}$$

where v is a real positive quantity. By substituting $\langle\Phi\rangle$ in $V(\Phi)$ one gets the minimum of the energy density for $v = \sqrt{\mu^2/\lambda}$, and around the minimum, in the *unitary gauge*, the Higgs doublet reads

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{1.12}$$

$H(x)$ being a real scalar field.

The value of v is known, because the first striking effect of symmetry breaking is that three gauge bosons become massive, the charged $W^\pm$ and Z. In particular, $m_W = gv/2 = m_Z \cos\theta_W$. From the experimental value of the Fermi constant G_F – see the next section – we get $v \sim 246$ GeV.

Substituting (1.12) in the Yukawa couplings reported in the last three lines of Eq. (1.1), we see how fermion mass terms are produced after the symmetry breaking. Let us consider, for example, the term of $\mathcal{L}$ coupling leptons with the Higgs field,

$$\mathcal{L}_{\mathrm{H},L} = -Y'^{l}_{\alpha\beta}\, \overline{L'_{\alpha L}}\, \Phi\, l'_{\beta R} + \mathrm{h.c.} \tag{1.13}$$

Once the Higgs field is developed around its minimum, one gets

$$\mathcal{L}_{\mathrm{H},L} = -\frac{v + H(x)}{\sqrt{2}} \left(Y'^{l}_{\alpha\beta}\, \overline{l'_{\alpha L}}\, l'_{\beta R} + \mathrm{h.c.} \right). \tag{1.14}$$

The term of Eq. (1.14) proportional to the vev provides the mass term for charged leptons, whereas the contribution proportional to $H(x)$ accounts for the trilinear coupling between charged leptons and the scalar boson H. Because the couplings Y'^{l} are generally not diagonal in the three-generations space, one must diagonalize them before interpreting (1.14) as a genuine mass term. A generic complex matrix such as Y'^{l} can be transformed into a diagonal form Y^{l} through a biunitary transformation

$$V_L^{l\dagger}\, Y'^{l}\, V_R^{l} = Y^{l} \qquad Y^{l}_{\alpha\beta} = y^{l}_{\alpha}\, \delta_{\alpha\beta}. \tag{1.15}$$

With the transformed leptonic fields defined as

$$l_R = V_R^{l\dagger} l'_R \quad \text{and} \quad l_L = V_L^{l\dagger} l'_L, \tag{1.16}$$

the term $\mathcal{L}_{\mathrm{H},L}$ can be rewritten as

$$\mathcal{L}_{\mathrm{H},L} = -\frac{v + H(x)}{\sqrt{2}} \sum_{\alpha} y_{l\alpha} \left(\overline{l_{\alpha L}}\, l_{\alpha R} + \mathrm{h.c.} \right). \tag{1.17}$$

From (1.17) one gets $m_\alpha = y^{l}_{\alpha}\, v/\sqrt{2}$ for $\alpha = e, \mu, \tau$. In terms of these masses one can also rewrite the interaction term between charged leptons and $H(x)$ as

$$\mathcal{L}^{I}_{\mathrm{H},L} = -H(x) \sum_{\alpha} \frac{m_\alpha}{v} \left(\overline{l_{\alpha L}}\, l_{\alpha R} + \mathrm{h.c.} \right), \tag{1.18}$$

which simply states that a heavier lepton is more strongly coupled to the Higgs field than a lighter one.

When the weak charged current $J^{\mu}_{\mathrm{W},L}$ is rewritten in terms of mass eigenstates $l_{\alpha L}$,

$$J^{\mu}_{W,L} = 2\, \overline{\nu'_{\alpha L}}\, (V_L)_{\alpha\beta}\, \gamma^{\mu}\, l_{\beta L}. \tag{1.19}$$

As long as neutrinos do not receive any mass term by the Higgs mechanism, because no right-handed partners ν_R have been introduced so far, we can redefine the neutrino field as $\nu_L = V_L^{\dagger}\,\nu'_L$ and get

$$J^{\mu}_{W,L} = 2\,\overline{\nu_{\alpha L}}\,\gamma^{\mu}\,l_{\alpha L}, \tag{1.20}$$

where by definition $l_{\alpha L} \equiv (e^-_L, \mu^-_L, \tau^-_L)$ and $\nu_{\alpha L} \equiv (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})$.

Concerning $J^{\mu}_{Z,L}$ of Eq. (1.7), one can easily see that by virtue of the unitarity of the matrices V^l_L and V^l_R it remains unchanged; hence one can simply replace the *primed* fields with the *unprimed* ones in (1.7).

For the Yukawa terms for quark fields one proceeds in the very same way. In the unitary gauge

$$\mathcal{L}_{\mathrm{H},Q} = -\frac{v + H(x)}{\sqrt{2}}\left(Y'^{D}_{\alpha\beta}\,\overline{q'^{D}_{\alpha L}}\,q'^{D}_{\beta R} + Y'^{U}_{\alpha\beta}\,\overline{q'^{U}_{\alpha L}}\,q'^{U}_{\beta R} + \mathrm{h.c.}\right). \tag{1.21}$$

Thus, by simultaneously diagonalizing the matrices of Yukawa couplings $(Y'^{D})_{\alpha\beta}$ and $(Y'^{U})_{\alpha\beta}$ via biunitary transformations V^D_L, V^D_R, V^U_L and V^U_R and defining the transformed quark fields as

$$q^D_R = V^{D\dagger}_R q'^D_R\,, \qquad q^D_L = V^{D\dagger}_L q'^D_L \tag{1.22}$$

$$q^U_R = V^{U\dagger}_R q'^U_R\,, \qquad q^U_L = V^{U\dagger}_L q'^U_L\,, \tag{1.23}$$

we can rewrite the mass term in $\mathcal{L}_{\mathrm{H},Q}$ as

$$\begin{aligned}\mathcal{L}_{\mathrm{H},Q} = &- \sum_{\alpha=d,s,b} m_{\alpha}\left(1 + \frac{H(x)}{v}\right)\overline{q^D_{\alpha L}}\,q^D_{\alpha R} \\ &- \sum_{\alpha=u,c,t} m_{\alpha}\left(1 + \frac{H(x)}{v}\right)\overline{q^U_{\alpha L}}\,q^U_{\alpha R} + \mathrm{h.c.}\end{aligned} \tag{1.24}$$

In this case, however, as all quarks are massive and have different masses, we have no freedom to arbitrarily rotate D or U quarks in the hadronic weak charged current

$$J^{\mu}_{W,Q} = 2\overline{q^U_{\alpha L}}\gamma^{\mu}\left(V^{U\dagger}_L V^D_L\right)_{\alpha\beta} q^D_{\beta L}. \tag{1.25}$$

The unitary matrix

$$V \equiv V^{U\dagger}_L V^D_L \tag{1.26}$$

is the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix (Cabibbo, 1963; Kobayashi and Maskawa, 1973), which depends upon three angles θ_{12} (the Cabibbo angle, θ_C, up to very small corrections), θ_{23} and θ_{13} and one phase δ,

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.27}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and $0 \leq \theta_{ij} \leq \pi/2$. To see that V can always be reduced to this form one has to recall that an arbitrary 3×3 unitary matrix has nine real parameters, but we have to subtract five free parameters connected with the single and independent rephasing of quark fields (the global rephasing still remains a symmetry of the system). In the case of quarks the three mixing angles satisfy a hierarchical structure, $s_{12} = 0.22535 \pm 0.00065$ (Beringer *et al.*, 2012), $s_{23} \sim s_{12}^2$, $s_{13} \sim s_{12}^4$.

As can be proven by studying the CP transformation (charge conjugation and parity) of the SM Lagrangian density written in terms of mass eigenstates, the only possible source of CP violation is encoded in the presence of the complex phase δ in V (1.27). Because CP violation processes have been observed in the hadron phenomenology, this has been ascribed to the presence of a nonvanishing value of δ. Indeed, the preferred value for this parameter is $\sin\delta \sim 0.93$ (Beringer *et al.*, 2012). All CP-violating effects in the quark sector can be expressed in terms of a single parameter, the Jarlskog parameter, which is invariant under the phase convention of quark fields:

$$J = -\mathrm{Im}\left[V_{us} V_{cd} V_{cs}^* V_{ud}^*\right] = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} s_\delta \sim 3 \times 10^{-5}. \tag{1.28}$$

In complete analogy to the leptonic case, one can show that unitarity of the matrices V_L^D, V_L^U, V_R^D, V_R^U implies that the expression for $J_{Z,Q}^\mu$ remains the same after the primed quark fields are replaced with the mass eigenstates.

1.3 The basic properties of neutrinos: interactions, masses and oscillations

1.3.1 *Neutrino interactions in the low energy limit*

The two terms in the SM Lagrangian density, $\mathcal{L}_I^{(\mathrm{CC})}$ and $\mathcal{L}_I^{(\mathrm{NC})}$ – see (1.4) and (1.6), respectively – describe a three-body process involving two fermions and $W^\pm$ and Z gauge bosons, whose mass is on the order of 100 GeV. Whenever the typical range for energies and momenta carried by the leptons (or quarks) is much smaller than this value, the gauge bosons produced in the trilinear vertex can only propagate as virtual particles. We will see that this is, for example, typically the case in almost all relevant cases in cosmology in which we will be interested in the following. The gauge propagators

$$\begin{aligned}
G_{\mu\nu}^W(x-x') &\equiv \langle 0|\mathrm{T}\, W_\mu(x) W_\nu^\dagger(x')|0\rangle = \lim_{\epsilon\to 0} i \int \frac{d^4p}{(2\pi)^4} \frac{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_W^2}}{p^2 - m_W^2 + i\epsilon} e^{-ip\cdot(x-x')} \\
G_{\mu\nu}^Z(x-x') &\equiv \langle 0|\mathrm{T}\, Z_\mu(x) Z_\nu^\dagger(x')|0\rangle = \lim_{\epsilon\to 0} i \int \frac{d^4p}{(2\pi)^4} \frac{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_Z^2}}{p^2 - m_Z^2 + i\epsilon} e^{-ip\cdot(x-x')}
\end{aligned} \tag{1.29}$$

can then be considered in their short-range limit,

$$G^W_{\mu\nu}(x-x') \xrightarrow{p^\mu \ll m_W} i\,\frac{g_{\mu\nu}}{m_W^2}\,\delta^4(x-x') \tag{1.30}$$

$$G^Z_{\mu\nu}(x-x') \xrightarrow{p^\mu \ll m_Z} i\,\frac{g_{\mu\nu}}{m_Z^2}\,\delta^4(x-x'). \tag{1.31}$$

Hence, the weak charged-current and neutral-current processes at tree level in the low energy limit are described by the effective Lagrangians

$$\mathcal{L}^{(\mathrm{CC})}_{\mathrm{eff}} = -\frac{g^2}{8m_W^2} J^{\mu\dagger}_W J_{\mu W} = -\frac{G_{\mathrm{F}}}{\sqrt{2}} J^{\mu\dagger}_W J_{\mu W} \tag{1.32}$$

$$\mathcal{L}^{(\mathrm{NC})}_{\mathrm{eff}} = -\frac{g^2}{4\cos^2\theta_W m_Z^2} J^{\mu\dagger}_Z J_{\mu Z} = -2\frac{G_{\mathrm{F}}}{\sqrt{2}}\rho J^{\mu\dagger}_Z J_{\mu Z}, \tag{1.33}$$

where $G_{\mathrm{F}} \equiv \sqrt{2}g^2/(8m_W^2) = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant and $\rho \equiv m_W^2/(m_Z^2\cos^2\theta_W)$, which is equal to unity in the SM.

The interaction terms $\mathcal{L}^{(\mathrm{CC})}_{\mathrm{eff}}$ and $\mathcal{L}^{(\mathrm{NC})}_{\mathrm{eff}}$ can mediate a set of purely four-lepton processes such as the ones reported in Tables 1.5 and 1.6. In the following we will treat only some of them in detail, but we will show how to use the results contained in Tables 1.5 and 1.6 for a simple generalization to all the others.

Let us consider in particular neutrino–electron elastic scattering, $\nu_e + e^- \to \nu_e + e^-$ and $\nu_{\mu(\tau)} + e^- \to \nu_{\mu(\tau)} + e^-$. From Eqs. (1.32) and (1.33) one can easily get the amplitudes

$$\mathcal{A}_{\nu_x e^- \to \nu_x e^-} = -\frac{G_{\mathrm{F}}}{\sqrt{2}}[\overline{u}_{\nu_x}\gamma^\rho(1-\gamma_5)u_{\nu_x}][\overline{u}_e\gamma_\rho(g_V^l - g_A^l\gamma_5)u_e] \tag{1.34}$$

$$\begin{aligned}\mathcal{A}_{\nu_e e^- \to \nu_e e^-} &= -\frac{G_{\mathrm{F}}}{\sqrt{2}}\left\{[\overline{u}_{\nu_e}\gamma^\rho(1-\gamma_5)u_e]\left\{[\overline{u}_e\gamma_\rho(1-\gamma_5)u_{\nu_e}]\right.\right.\\ &\quad \left.+\,[\overline{u}_{\nu_e}\gamma^\rho(1-\gamma_5)u_{\nu_e}][\overline{u}_e\gamma_\rho(g_V^l - g_A^l\gamma_5)u_e]\right\}\\ &= -\frac{G_{\mathrm{F}}}{\sqrt{2}}[\overline{u}_{\nu_e}\gamma^\rho(1-\gamma_5)u_{\nu_e}][\overline{u}_e\gamma_\rho((1+g_V^l)-(1+g_A^l)\gamma_5)u_e],\end{aligned} \tag{1.35}$$

where $x = \mu, \tau$, $g_V^l \equiv g_L^l + g_R^l$, and $g_A^l \equiv g_L^l - g_R^l$ (see Table 1.4). Note that to obtain the final form of $\mathcal{A}_{\nu_e e^- \to \nu_e e^-}$ we have used one of the Fierz rearrangement formulas. In Tables 1.5 and 1.6 are reported the squared amplitudes for several pure weak leptonic processes at three level. The list is not complete, but the missing processes can be obtained using crossing symmetry.